International Journal of Economics and Management Sciences
Volume 4, Issue 2 (2025), pp. 45-70

Optimization of Money Management by Stochastic Control: Theoretical Foundations and Practical Applications.

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Cite this article Fraraji,H., Elkabbouri,M., (2025). Optimization of Money Management by Stochastic Control: Theoretical Foundations and Practical Applications.
International Journal of Economics and Management Sciences, Volume 4, Issue 2 (2025), pp. 45-70

Abstract:

In a financial environment marked by increasing uncertainty and market complexity, the optimization of money management through stochastic control is emerging as a robust and adaptive approach to dynamic capital management. This article presents a comprehensive review of stochastic models commonly used to model financial asset dynamics, including geometric Brownian motion, jump processes and Lévy processes, as well as associated optimization methods such as expected utility maximization, risk minimization, and dynamic programming via the Hamilton-Jacobi-Bellman equation.

We then illustrate the concrete application of these tools in several financial contexts: pension fund management, commodity trading companies and algorithmic investment strategies. The analysis highlights the advantages of stochastic approaches in terms of flexibility, adaptability and risk control, while discussing their practical and theoretical limitations.

Finally, the article opens up promising perspectives towards the integration of machine learning techniques and environmental, social and governance (ESG) criteria for more efficient and responsible finance.

Keywords: Stochastic control; Money management; Dynamic optimization; Jump models; Portfolio management.

Introduction

In a financial environment characterized by uncertainty, complexity and volatility, rigorous capital management - or money management - is a key issue for institutional investors and algorithmic traders alike. Far from being a simple technique for adjusting positions, money management is now seen as a strategic lever for performance and risk control.

At the same time, advances in financial mathematics have led to the emergence of powerful theoretical frameworks for decision-making in uncertain environments. Among these, stochastic control occupies a major place. Heir to optimal control theory and probabilistic modeling, it offers sophisticated tools for formulating, analyzing and solving dynamic optimization problems in random environments, such as those of the financial markets.

The integration of stochastic control into the money management process enables the transition from a static logic to an adaptive management approach, where investment decisions are adjusted according to the continuous evolution of market conditions, regulatory constraints and the investor's objectives. In particular, this approach enables us to:

- Formalize price dynamics using realistic stochastic models (geometric Brownian motion, jump processes, Lévy processes),
- Optimize decisions through utility, risk or adjusted return functions,
- Integrate operational constraints into a coherent mathematical framework,
- Apply these methods in concrete contexts (pension funds, commodity trading, algorithmic strategies).

The aim of this article is to provide a structured, in-depth overview of the application of stochastic control to money management. After presenting the main stochastic models used to model market dynamics, we introduce the associated optimization tools (utility maximization, risk minimization, dynamic programming, etc.), then illustrate their concrete application through several real-life, operational case studies.

By combining mathematical rigor and practical relevance, this work aims to enlighten financial decision-makers, quantitative engineers and researchers on the benefits and limits of the stochastic approach applied to capital management in the era of complex markets and massive data.

1. Stochastic modeling of market dynamics

Stochastic modeling is at the heart of quantitative finance. It provides a realistic representation of the random evolution of asset prices, taking into account continuous volatility, price jumps and the time structure of returns. Three main families of models can be distinguished: geometric Brownian motion (GBM), Lévy processes (with or without jumps), and reversionary processes such as Ornstein-Uhlenbeck.

1.1. Geometric Brownian Motion (GBM)

The GBM is the basic model for continuous-time price dynamics. Described by :

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where:

- S_t is the asset price at time ttt,
- μ is the drift term (expected return rate),
- σ is the volatility term (standard deviation of returns),
- W_t is a standard Brownian motion (Wiener process),
- dt is an infinitesimal increment in time.

it assumes lognormal returns, constant volatility and a continuous trajectory. It forms the basis of the Black-Scholes model (Black & Scholes, 1973) and is still used for option pricing, market scenario simulation and risk/return modeling in portfolio models.
However, it fails to capture the sharp jumps and thick tails observed in reality.

1.2. Lévy and jump processes

To model price discontinuities, Lévy processes generalize the GBM by incorporating both a continuous (Brownian) component and random jumps. They are characterized by their Lévy-Khintchine function ψ and include :

$$E\left[e_t^{iuX}\right] = e^{t\psi u}$$

where the Lévy exponent ψ(u)\psi(u)ψ(u) has the form:

$$\psi(u) = iau - 12\sigma^2 u^2 + \int_{R\setminus\{0\}} (e^{iux} - 1 - iux1_{|x|<1})\nu(dx).$$

where:

- $a \in R$ is a drift term,
- $\sigma \geq 0$ is the volatility of the Brownian component,

- $\boldsymbol{\nu}$ is the Lévy measure that controls the jump intensity and jump size distribution; it satisfies $\int (\mathbf{1} \wedge \boldsymbol{x}\text{^}\mathbf{2})\boldsymbol{\nu}(\boldsymbol{dx}) < \infty$,
- The integral term captures the jump behavior, with a compensation term $\boldsymbol{iux1}_{|x|<1}$ for small jumps.

- The compound Poisson process,
- The Variance Gamma (VG) model (Madan & Seneta, 1990),
- The CGMY process (Carr, Geman, Madan, & Yor, 2002), which allows flexibility in the shape of the distribution tails.

These models offer a better empirical fit to financial return distributions, notably by capturing asymmetry and leptokurtia.

1.3. Reversion process: Ornstein-Uhlenbeck

The Ornstein-Uhlenbeck (OU) process is a stationary Gaussian diffusion, suitable for variables with a tendency to revert to a mean value (reversion):

$$dX_t = \theta(\mu - X_t), dt + \sigma \, dW_t$$

where:

- X_t is the state variable at time ttt,
- $\theta > 0$ is the speed of mean reversion, controlling how fast X_t tends to return to the mean,
- $\mu \in R$ is the **long-term mean level**,
- σ>0 is the volatility coefficient,
- *W_t* is a standard Brownian motion.

It is commonly used to model interest rates (Vasicek, 1977) , stochastic volatility (Heston, 1993) , and commodity prices. Unlike the GBM, the OR can model long-term stability, although it remains limited by its linearity and inability to handle jumps or multiplicative effects.

Table 1: Comparative Summary of Stochastic Models in Financial Applications

Model	Kind	Features	Benefits	Boundaries
GBM	Continuous broadcast	Constant volatility, continuous trajectories	Simplicity, analytical basis, positives	Does not model jumps or thick tails
Merton (jumps)	Diffusion + Fish	Rare jumps, parametric structure	Capturing market shocks	Complex calibration

Lévy process	Generalization	Broadcast + frequency hopping and various sizes	High flexibility, empirical adequacy	Numerical complexity and estimation
Ornstein - Uhlenbeck	Mean reversion	Linear model, return to equilibrium	Realism on rates/volatility, stationarity	Negativity possible, no jumps

2. Stochastic optimization of investment decisions

In an uncertain environment, dynamic portfolio management is based on principles derived from stochastic control, enabling allocation decisions to be optimized according to expected performance and risk incurred. This section summarizes the classical approaches: utility maximization, risk minimization, constraint integration and risk-adjusted performance evaluation.

2.1. Maximizing expected utility

Utility theory provides a fundamental framework in which the investor seeks to maximize:

$$\max_{\pi_t} \mathbb{E}[U(W_T)]$$

where:

- W_T is the wealth at the terminal time T,
- π_t denotes the amount invested in the risky asset at time ttt,
- $U(\cdot)$ is the investor's utility function reflecting risk preferences.
- Portfolio dynamics generally follow :

$$dW_t = [r\,W_t + \pi_t(\mu - r)]dt + \pi_t \sigma dW_t$$

Common utility functions include :

- Logarithmic: favors capital growth,
- Exponential (CARA): constant risk tolerance,
- CRRA (isoelastic): decreasing marginal return.

Merton's (1971) solution for a CRRA investor gives: (Merton, 1971)

$$\pi^* = \frac{\mu - r}{\gamma\sigma^2}$$

This framework allows customization according to risk aversion, but is highly dependent on market parameters, which are often difficult to estimate.

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Volume 4, Issue 2 (2025), pp. 45-70
ISSN 2823-9350

2.2. Risk minimization (Markowitz approach)

Markowitz (1952) proposes quadratic portfolio optimization: (Markowitz, 1952)

$$\min_{w} \; w^{\top} \Sigma w \quad subject\ to \begin{cases} w^{\top} \mu = \bar{r} \\ w^{\top} \mathbf{1} = 1 \end{cases},$$

where:

- $w \in R^n$: vector of portfolio weights,
- $\mu \in R^n$: vector of expected asset returns,
- $\Sigma \in R^{n \times n}$: covariance matrix of asset returns,
- $\bar{r} \in R$: target expected return,
- $\mathbf{1} \in R^n$: vector of ones (to enforce full investment).

This approach is used to construct the efficient frontier: portfolios that maximize returns for a given level of risk. It remains a standard for ex ante risk analysis, although it :

- Relies on precise estimation of μ and Σ,

- Neglects asymmetries and extremes,

- Does not distinguish between beneficial and harmful volatility.

2.3. Taking constraints into account

In practice, the investor is subject to multiple constraints:

- Budgetary: $\sum w_i = 1$,

- Non-negativity: $w_i \geq 0$,

- Sectors or regulatory limits,

- Liquidity and position size.

These constraints transform optimization into constrained quadratic problems, solved by numerical methods (simplex, interior points, convex optimization). (Boyd & Vandenberghe, 2004)

2.4. Risk-adjusted performance measures

The evaluation of a strategy is not based solely on absolute return, but on its effectiveness relative to the risk taken. Several indicators are used (Sharpe, 1966):

- Sharpe ratio :

$$S = \frac{R_p - R_f}{\sigma_p}$$

where:

- R_p : expected return of the portfolio

- $\boldsymbol{R_f}$: risk-free rate
- $\boldsymbol{\sigma_p}$: standard deviation of portfolio returns (total volatility)

- Treynor's ratio (systematic risk via beta),
- Jensen's Alpha: measures outperformance relative to the CAPM model,
- Sortino's ratio: only takes negative volatility into account.

These indicators are key to objectively comparing several portfolios, taking into account the risks involved.

Table 2: Visual Summary of Portfolio Optimization Approaches

Approach	Objective	Benefits	Boundaries
Expected utility	Maximize expected satisfaction	Personalization, rigorous framework	Strong reliance on assumptions
Variance minimization	Reduce total risk	Simplicity, standard framework	Neglects asymmetry and extremes
Allocation constraints	Reflect actual restrictions	Operational realism	Increase in digital complexity
Risk-adjusted ratios	Normalized comparison	Easy to interpret	Do not cover all aspects of real risk

3. Applications of stochastic control to money management

Stochastic control provides a rigorous framework for dynamic portfolio management. It allows us to model the uncertainty of financial markets while optimizing decisions in continuous time. This section summarizes its most striking operational applications.

3.1. Dynamic rebalancing

Rebalancing consists in periodically adjusting the composition of a portfolio to maintain a target allocation. In a stochastic framework, this strategy aims to minimize deviations from the target while controlling transaction costs: (Davis & Norman, 1990)

$$\min_{w_t} E\left[\int_0^T \left(\| w_t - w^* \|^2 + C(w_t, w_{t^-})\right) dt\right]$$

where:

- w_t is the portfolio allocation at time ttt,
- w^* is the target (optimal or benchmark) allocation,
- $\boldsymbol{C(w_t, w_{t^-})}$ represents transaction costs incurred from adjusting positions,
- $\| \boldsymbol{w_t} - \boldsymbol{w^*} \|^2$ penalizes deviation from the target,
- $\boldsymbol{W_{t^-}}$ is the allocation just before time t (left limit, to capture jumps or discrete adjustments).

Approaches such as that of Davis & Norman (1990) integrate these constraints into a dynamic programming formulation, leading to an optimal strategy that takes into account both market fluctuations and operational costs.

3.2. Arbitrage strategies

Stochastic control makes it possible to formalize and optimize arbitrage strategies, which exploit temporary price inefficiencies: (Bellman, 1957)

$$\boldsymbol{max}\max_{\pi_t} \boldsymbol{E}\left[\int_0^T \boldsymbol{\pi}_t^\top \boldsymbol{dSt} - \boldsymbol{Co\hat{u}ts}(\boldsymbol{\pi}_t)\right]$$

where:

- π_t vector of positions (e.g., long/short in various assets),
- $\boldsymbol{S_t}$: price process of the assets involved,
- $\boldsymbol{Co\hat{u}ts}(\boldsymbol{\pi_t})$: function capturing trading frictions (e.g., impact, commissions, slippage),

These techniques apply to statistical, inter-market or volatility arbitrage. The Hamilton-Jacobi-Bellman (HJB) equation helps determine the optimal policy in the presence of execution costs and risks.

3.3. Dynamic programming in finance

Dynamic programming is based on Bellman's (1957) optimality principle. It solves portfolio problems by modeling the value function V(t,x)V(t, x)V(t,x) through the equation HJB : (Bellman, 1957)

$$\frac{\partial V}{\partial t} + \sup_{\pi}\{L^\pi V + f(t, x, \pi)\} = 0$$

where:

- $\boldsymbol{\pi}$: control variable (e.g., investment strategy),
- $\boldsymbol{L^\pi}$: infinitesimal generator of the controlled stochastic process (includes drift and diffusion terms),
- $\boldsymbol{f(t, x, \pi)}$: running cost or utility function (e.g., consumption, arbitrage gain),
- $\boldsymbol{V(T, x) = g(x)}$: terminal condition, often representing utility from terminal wealth.

This approach is suitable for both portfolio management and dynamic hedging, but often requires numerical resolutions (finite differences, Monte Carlo).

3.4. Strategic interactions: Game theory

Stochastic game theory models interactions between several investors whose strategies influence each other. It is useful for : (Basar & Olsder, 1999)

- Representing competition between institutional players,
- Study market strategies in an uncertain environment,
- Formalize collective bargaining or hedging situations.

Stochastic Nash equilibria are deduced from coupled HJB systems.

3.5. Hedging and risk management

Dynamic hedging techniques (delta, gamma, vega hedging) can be optimized via stochastic control. The aim is to minimize exposure to residual risk while taking into account readjustment costs. (Hull, 2012)

These strategies can be applied to

- Foreign exchange risk management,
- Protection of bond portfolios,
- Exotic options.

Table 3: Summary of Advanced Applications of Stochastic Control in Finance

Application	Objective	Benefits	Boundaries
Rebalancing	Maintaining the target allocation	Dynamics, cost management	Complexity, transaction costs
Arbitration	Exploiting market inefficiencies	Opportunistic profitability, rigorous framework	Calibration, slippage
Dynamic programming	Sequential optimization	Adaptive strategies, theoretical robustness	Numerical difficulty in high dimension
Game theory	Anticipation of competing behaviors	Models competition and cooperation	Complex HJB systems, strong assumptions

4. Portfolio management models and practices

Modern portfolio management is based on rigorous optimization models, integrating the notion of diversification, dynamic capital management and differentiated strategies according to performance objectives. This section presents three key approaches: the Markowitz model, Kelly's strategy, and the debate between active and passive management.

4.1. The Markowitz model and efficient diversification

The Markowitz model (1952) is the foundation of modern portfolio theory. It proposes optimizing the composition of a portfolio by minimizing its variance for a given expected return, thereby tracing the efficient frontier of non-dominated portfolios.

Formally : (Markowitz, 1952)

$$\min_{w} \; w^{\top} \Sigma w \quad \textit{subject to} \begin{cases} w^{\top} \mu = \bar{r} \\ w^{\top} 1 = 1 \end{cases},$$

This theoretical framework shows that diversification - the combination of weakly or negatively correlated assets - reduces specific risk, thus improving the risk/return trade-off.

Limitations :

- Assumption of normality of returns,
- Highly sensitive to errors in estimating μ\muμ and Σ\SigmaΣ,
- Solutions sometimes unstable.

Extensions such as CAPM, robust models and Bayesian methods have been proposed to remedy these shortcomings.

4.2. Kelly's strategy: Optimum logarithmic growth

Kelly's (1956) formula proposes maximizing the expectation of logarithmic capital growth, with a view to long-term performance:

$$\max_{f} \mathrm{E} \; [\log(\mathrm{WT})]$$

where f is the fraction of capital invested, and W_T is terminal wealth. Under simple probabilistic returns (e.g., a binary gamble), the formula yields the fraction f^* of capital to bet:

$$f^* = \frac{p \cdot b - q}{b},$$

where:

- P : probability of winning,
- $q \; = 1 - p$,
- b: payout ratio (gain per unit bet).

It provides an optimal allocation of capital based on probability of gain and expected returns. In a multi-asset context, we seek to maximize $E[\log(w^{\top} R)]$ under budgetary constraint. (Kelly, 1956)

Advantages:

- Asymptotically optimal strategy,

- Highly consistent with a long-term objective,

- Suitable for algorithmic trading or systematic management.

Limitations:

- Extremely sensitive to estimation errors,

- Risk of overoptimization in the event of overestimation of return or underestimation of risk.

Variants such as the Kelly fractional are used to reduce volatility.

4.3. Active vs. passive management: complementary approaches

There are two opposing philosophies in portfolio management:

Passive management

- Objective: replicate an index (e.g. S&P 500),

- Advantages: low fees, transparency, market efficiency,

- Limitations: no outperformance, blind exposure to all market phases.

Active management

- Objective: generate alpha by exploiting market inefficiencies,

- Advantages: flexibility, adaptability,

- Limitations: high costs, uncertain results, need for constant expertise.

Hybrid strategies

Many managers adopt a mixed approach, combining :

- A passive core for broad market exposure,

- An active pocket targeting specific opportunities.

This approach balances stability, cost and personalization. (Fama & French, 2010)

Table 4: Visual Synthesis of Portfolio Allocation and Management Strategies

Approach	Objective	Benefits	Boundaries
Markowitz	Minimize risk for return	Diversification, analytical framework	Sensitive estimate, strong assumptions
Kelly	Maximize logarithmic growth	Long-term performance, probabilistic rigor	Over-optimization, high potential volatility
Passive management	Replicate the market at lower cost	Low costs, simplicity	No control or overperformance possible
Active management	Beat the market by selection	Adaptability, potential alpha generation	High costs, uncertainty of results

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ISSN 2823-9350

4.4. Visual Overview of Stochastic Control in Financial Optimization

To enhance understanding of the stochastic control framework applied to portfolio optimization, we present a schematic representation of the key steps involved in the process. This diagram provides a conceptual flow from the mathematical modeling of asset dynamics to the determination of optimal investment strategies via dynamic programming techniques.

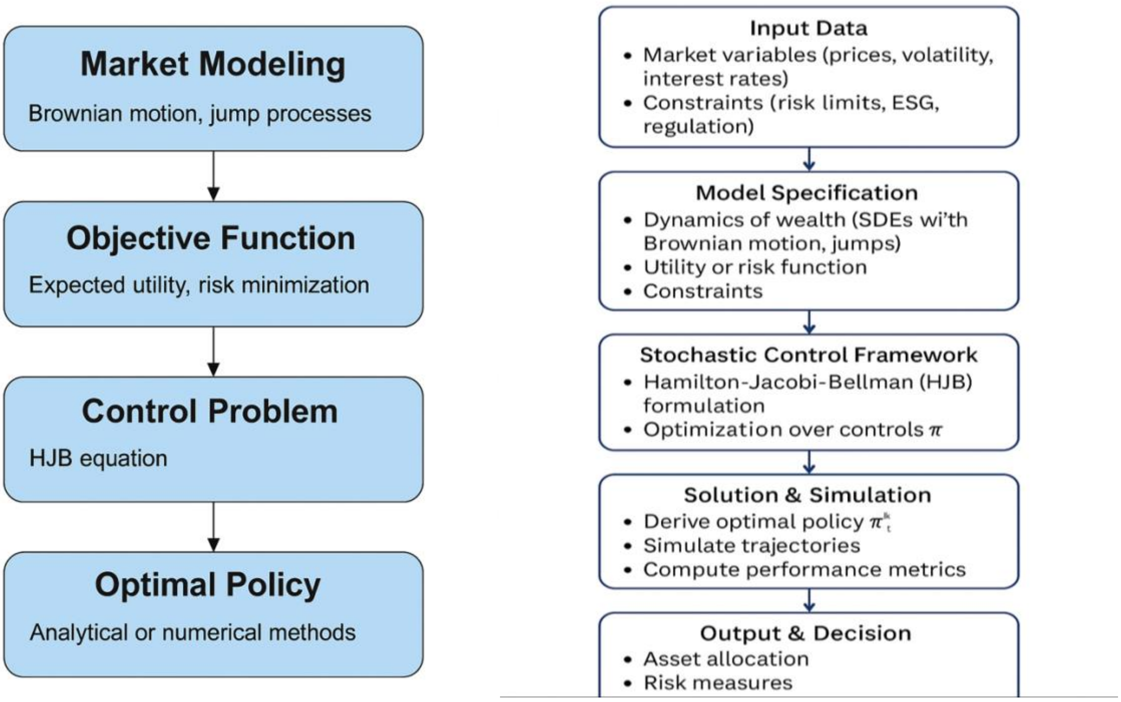

Figure 1 : Stochastic Control Flow in Financial Optimization

This flowchart illustrates the essential stages of the stochastic control process: starting with market modeling (Brownian motion, jump processes), then defining the objective function (e.g., expected utility, risk minimization), formulating the control problem (e.g., via the Hamilton-Jacobi-Bellman equation), and finally solving for the optimal policy using analytical or numerical methods.

This structure supports the formulation of investment strategies that are not static but dynamically adjusted over time, reflecting changes in market states and investor preferences. The flowchart emphasizes the recursive nature of stochastic decision-making and the central role of the value function in capturing future expectations and risks.

4.5. Numerical Illustration: Simulated Wealth Paths under Merton's Optimal Strategy

To concretely illustrate the application of stochastic control in portfolio optimization, we simulate the evolution of investor wealth under Merton's optimal allocation strategy. This

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ISSN 2823-9350

classical model, based on maximizing expected utility under constant relative risk aversion (CRRA), yields a closed-form expression for the optimal proportion of wealth to be invested in the risky asset:

$$\pi^* = \frac{\mu - r}{\gamma\sigma^2}$$

Parameters Used:

- $\mu = 8$ %: Expected return of the risky asset
- $r = 2\%$: Risk-free rate
- $\sigma = 15\%$: Volatility of the risky asset
- $\gamma = 3$: Risk aversion coefficient
- $T = 1$: Investment horizon (1 year)
- $W_0 = 1$: Initial wealth
- $\Delta t = 1/252$: Daily steps (252 trading days)

Python Simulation Code:

```
import numpy as np
import matplotlib.pyplot as plt
⇒ Parameters
mu, r, sigma, gamma = 0.08, 0.02, 0.15, 3
pi_star = (mu - r) / (gamma * sigma**2)
W0, T, N = 1, 1.0, 252
dt = T / N
n_paths = 10
⇒ Simulation
np.random.seed(42)
time_grid = np.linspace(0, T, N)
trajectories = np.zeros((n_paths, N))
trajectories[:, 0] = W0
for i in range(n_paths):
    for t in range(1, N):
        dWt = np.random.normal(0, np.sqrt(dt))
        drift = (r + pi_star * (mu - r)) * trajectories[i, t-1]
        diffusion = pi_star * sigma * trajectories[i, t-1] * dWt
        trajectories[i, t] = trajectories[i, t-1] + drift * dt + diffusion
```

⇒ **Plot**

```
plt.figure(figsize=(10, 6))
for i in range(n_paths):
   plt.plot(time_grid, trajectories[i])
plt.title("Simulated Wealth Trajectories $W_t$ under Merton's Optimal Strategy")
plt.xlabel("Time (years)")
plt.ylabel("Wealth $W_t$")
plt.grid(True)
plt.tight_layout()
plt.show()
```

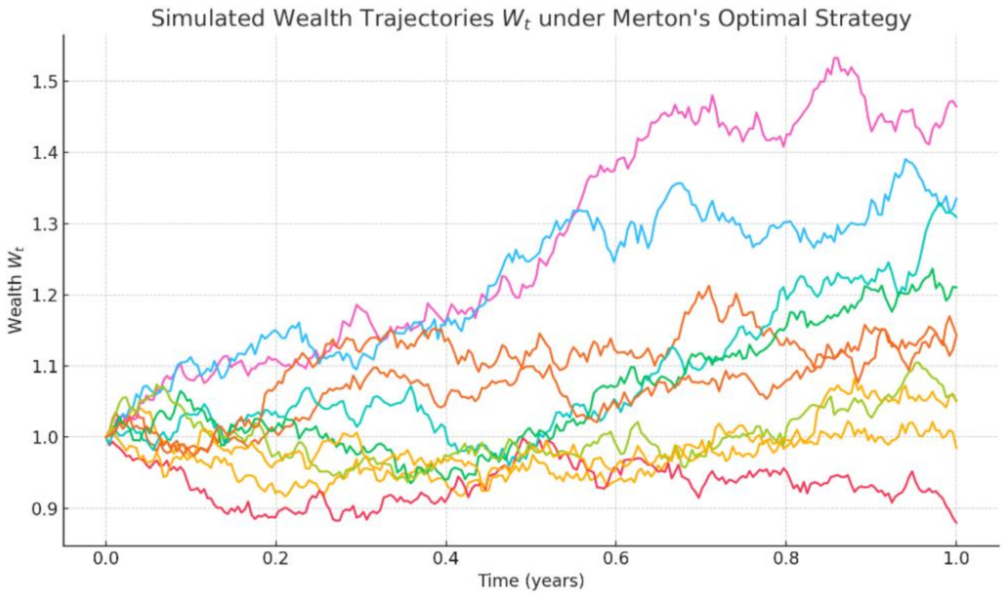

Figure 2 : Simulated Wealth Trajectories W_t under Merton's Optimal Strategy.

Interpretation:

The plot shows 10 simulated paths of investor wealth over one year, assuming the investor follows the constant optimal allocation π^* derived from Merton's formula. Although the initial wealth is the same across all paths, the randomness inherent in the market (modeled as a Brownian motion) leads to divergence in final outcomes. The trajectories remain upward-trending on average, reflecting the positive expected return, but they also exhibit variability due to market volatility.

This simulation helps validate theoretical results and provides an intuitive understanding of the dynamic behavior of wealth under optimal stochastic control strategies.

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This simulation illustrates a basic yet operational implementation of Merton's optimal strategy using Python. The full source code, included above, provides a practical framework for testing stochastic control models under different assumptions and can be extended to include transaction costs, constraints, or multi-asset settings.

5. Sensitivity analysis in financial optimization

Sensitivity analysis is a critical tool in assessing the robustness of money management strategies optimized by stochastic control. It measures the impact of uncertainties in input parameters - such as expected returns, volatility or correlations - on allocation decisions.

5.1. Objectives

- Assess the robustness of portfolios to estimation errors,
- Identify parameters influencing performance or risk,
- Quantify model risk,
- Adapt strategies to market shocks. (Fabozzi et al., 2007)

5.2. Classical methods

- Partial derivatives: calculation of $\frac{\partial f}{\partial \theta}$ to estimate the marginal sensitivity of an objective function (utility, variance) to a parameter. (Meucci, 2005)
- Cross-sensitivities: $\frac{\partial^2 f}{\partial \theta_i \partial \theta_j}$, to assess non-linear interactions. (Glasserman, 2004)
- Parametric scenarios: ±10 to ±30% variation of key parameters to observe the effect on portfolio composition, Sharpe ratio or VaR.

5.3. Practical applications

- Stress testing: analysis of portfolios under extreme conditions (crisis, high volatility, interest-rate shocks).
- Robust backtesting: comparison of several model calibrations on historical data.
- Optimization under uncertainty: integration of sensitivity penalties in the objective function (so-called “robust” approach).

5.4. Limitations

- Approach often local, sensitive to assumed linearity around the optimum point.
- High computational cost for high-dimensional models.
- Results dependent on the method used (analytical differentiation vs. Monte Carlo simulation).[19]

6. Case studies : Practical applications of stochastic control

Stochastic control has many applications in asset management, in contexts as varied as pension funds, commodity trading and algorithmic investment. These cases illustrate the theoretical framework's ability to adapt to specific constraints and practical objectives.

6.1. Pension Funds: Long-Term Stability under Constraints

Pension funds are a key example of long-term asset management, where the challenge lies in ensuring the sustainability of future benefit payments while operating under strict regulatory frameworks and macroeconomic uncertainty. Their primary goal is to secure liabilities over extended horizons, while maintaining a controlled level of portfolio volatility and intergenerational equity.

Objectives

The central objective is to maximize the probability of fulfilling future liabilities while minimizing the intertemporal volatility of net assets. This must be achieved within regulatory allocation constraints (e.g., Solvency II) and amid uncertainties surrounding interest rates, inflation, and financial returns.

Methodology

Modern pension fund management is built on a quantitative architecture combining:

- Stochastic modeling of wealth W_t, integrating asset returns and liability cash flows (contributions, benefit payments);
- Liability-Driven Investment (LDI) strategies, which aim to align asset dynamics with actuarial liabilities;
- Adaptive rebalancing policies, adjusting the portfolio dynamically based on the funding ratio and market conditions, often modeled using stochastic control techniques.

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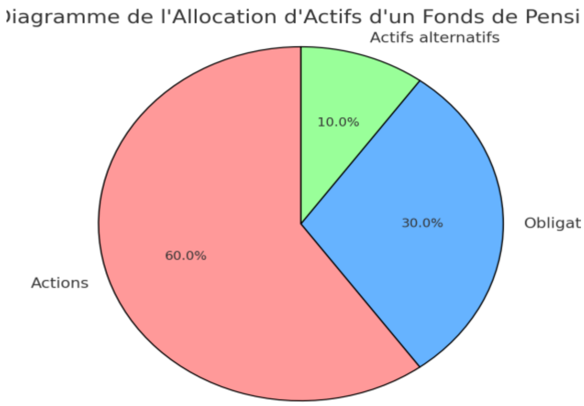

Figure 3 : Typical Strategic Asset Allocation of a Pension Fund

The pie chart shows a typical long-term allocation strategy for a defined benefit pension fund: 60% in equities, 30% in fixed-income securities (bonds), and 10% in alternative assets such as real estate, infrastructure, or private equity.

This allocation aims to diversify return sources while ensuring long-term actuarial balance. Bonds are used to hedge liability cash flows, equities provide long-term risk premia, and alternative assets help smooth returns across market cycles.

Strategic Interpretation

This allocation reflects a robust diversification strategy based on the principles of stochastic optimization:

- Equity exposure captures long-term growth potential and risk premia needed to meet future obligations.
- Bond holdings serve to stabilize portfolio returns and align with predictable liability outflows.
- Alternatives introduce resilience and diversification, with return streams often decorrelated from traditional markets.

Practical Case Study

A pension fund with a 30-year investment horizon increased its equity allocation from 50% to 70% to capture higher expected returns. Simultaneously, it implemented put option overlays to hedge downside risk, within a stochastic dynamic management framework. This strategy exemplifies the application of adaptive money management, combining return optimization and active risk mitigation.

6.2. Integrated Risk Hedging in a Commodity Trading Firm

Commodity trading companies—whether in energy, metals, or agricultural products—operate in markets characterized by high price volatility, logistical cost fluctuations, and foreign exchange risks. In this context, the implementation of integrated hedging strategies constitutes a concrete application of stochastic control in risk management (Geman, 2005) .

Objective

The primary objective is to stabilize profit margins while dynamically managing both physical and financial positions. This ensures operational resilience and secures long-term profitability amid unstable market conditions(Lucia & Schwartz, 2002) .

Methodology: Stochastic Modeling and Hedging Strategies

The hedging strategy relies on three methodological pillars:

- Stochastic price modeling, using mean-reverting processes such as the Ornstein-Uhlenbeck process (Geman, 2005), or jump processes like Lévy models, which better capture market discontinuities frequently observed in commodity markets (Eydeland & Wolyniec, 2003).
- Use of derivative instruments such as futures, options, and commodity swaps, which allow companies to lock in future prices or exchange fixed and floating cash flows indexed to commodity prices (Lucia & Schwartz, 2002) .
- Stochastic optimization of expected margin, adjusted for logistical costs and price forecasts, implemented within an optimal control framework under uncertainty (Eydeland & Wolyniec, 2003) .

Results and Benefits

The implementation of these techniques yields several measurable benefits:

- A significant reduction in the volatility of financial results, improving earnings predictability (Geman, 2005) ;
- Enhanced logistical allocation, thanks to the integration of price forecasts in storage and shipment decisions (Lucia & Schwartz, 2002);

- Preserved operational solvency, a critical factor in capital-intensive and shock-sensitive markets (Eydeland & Wolyniec, 2003) .

Perspectives

This case demonstrates that stochastic control applied to commodity price risk management represents a strategic advantage for trading firms. It also opens up avenues for future research in:

- Incorporating complex regulatory constraints,
- Integrating exogenous macroeconomic signals,
- Exploring hybrid models combining machine learning and traditional stochastic methods.

6.3. Algorithmic Investment: Adaptive and Automated Optimization

Algorithmic investment refers to the use of mathematical models, artificial intelligence, and computational techniques to automate portfolio decisions and trading strategies. As financial markets grow increasingly complex and data-driven, algorithmic methods offer an efficient and scalable framework for responding to market dynamics in real time.

Objectives

The primary goal is to maximize risk-adjusted performance by automating decisions that are traditionally influenced by human biases, while reacting rapidly to evolving market conditions.

Methodological Framework

Algorithmic strategies are typically grounded in:

- Markov Decision Processes (MDP) and optimal stochastic control formulations for sequential decision-making;
- Stochastic models incorporating diffusion, jumps, or reinforcement learning (RL) to adapt to nonlinear and high-frequency dynamics;
- Implementation in quantitative trading, including statistical arbitrage, market making, and momentum-based execution strategies.

Key Strategy Types

- High-Frequency Trading (HFT): Leverages ultra-low latency infrastructure to exploit micro-inefficiencies in market pricing. Algorithms react in milliseconds to real-time order book data and execute trades to capture small yet frequent profits.

- Momentum Strategies: Identify and follow market trends by taking long positions on outperforming assets and short positions on underperformers. These strategies rely on technical indicators and are designed to capitalize on investor behavior and price persistence.
- Statistical Arbitrage: Exploits temporary pricing anomalies between correlated assets. Sophisticated models identify deviations from historical price relationships and execute paired trades to profit from mean reversion.

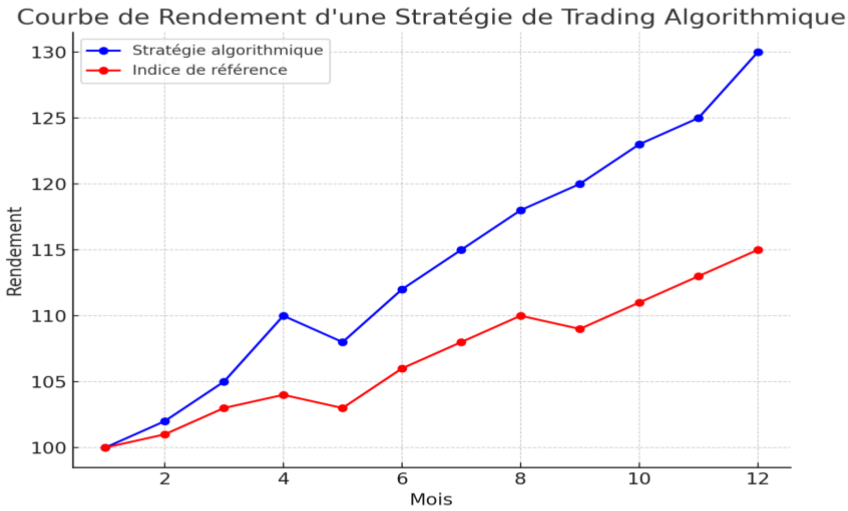

Figure 4 – Performance Curve of an Algorithmic Trading Strategy

The comparative performance of an algorithmically managed portfolio versus a benchmark index. The strategy uses real-time data and automated decision-making to deliver superior risk-adjusted returns.

Case Study

A quantitative investment fund deployed a machine learning-based trading model to forecast stock price movements. The model integrates thousands of features including financial indicators, macroeconomic news, and social media sentiment. By executing real-time trades based on model outputs, the fund consistently outperformed market benchmarks. This illustrates the transformative power of artificial intelligence in modern investment management, particularly in the processing of unstructured data and adaptive portfolio allocation.

6.4. Regulatory and ESG Implications in Stochastic Control-Based Asset Management

The application of stochastic control in money management increasingly intersects with growing regulatory oversight and the integration of Environmental, Social, and Governance (ESG) considerations. These dimensions are crucial for aligning advanced financial strategies with evolving compliance norms and sustainability objectives.

6.4.1. Regulatory Considerations in Algorithmic and Dynamic Trading

Financial regulators worldwide—such as the SEC (USA), ESMA (EU), and AMMC (Morocco)—have introduced frameworks that govern automated trading practices, capital requirements, and risk exposure disclosure. In the context of stochastic control, especially in high-frequency or dynamically rebalanced portfolios, several key regulatory aspects must be considered:

- Transparency and Explainability: Regulators increasingly require that algorithmic trading systems provide auditable logic. This challenges the use of complex models (e.g., reinforcement learning or deep stochastic control) whose decisions may lack interpretability.
- Stress Testing and Capital Adequacy: Models must include robust sensitivity analysis and stress testing, aligned with Basel III requirements and local regulations. Stochastic simulations provide tools for demonstrating capital resilience under extreme market conditions.
- Latency and Market Abuse: Controls must be in place to prevent market manipulation and ensure fair access. This affects how high-frequency implementations of stochastic optimization can be deployed.

6.4.2. ESG Integration into Stochastic Optimization

ESG investing is no longer a niche practice but a systemic requirement in portfolio management. Institutions are expected to demonstrate that their investment decisions account for ESG-related risks and impacts. In the stochastic control framework, this translates into:

- Objective Function Adjustment: Utility functions or cost functions can incorporate ESG penalties or scores, influencing optimal asset allocation. For example, portfolios can be penalized for allocating capital to low-rated ESG sectors.

- Constraint-Based Formulation: ESG guidelines can be introduced as hard constraints (e.g., minimum percentage of assets in sustainable bonds) or soft preferences within optimization models.
- Scenario Generation and ESG Stress Testing: Stochastic models can simulate climate risk, regulatory transition scenarios, or carbon pricing shocks to test portfolio sustainability and robustness.

Illustrative Example

Let π_t be the vector of dynamic allocations across assets, and ESG_t be the vector of ESG scores. A modified stochastic control objective could be:

$$\max_{\pi_t} E\left[\int_0^T U(Wt) - \lambda \cdot Penalty_{ESG}(\pi_t, ESG_t)\, dt\right]$$

where λ reflects the investor's ESG sensitivity, and $Penalty_{ESG}$ quantifies ESG underperformance.

Integrating regulatory constraints and ESG factors into stochastic control frameworks ensures that money management strategies are not only quantitatively robust but also compliant and socially responsible. These dimensions are essential for institutional acceptance and long-term investment legitimacy.

Conclusion

The convergence of money management and stochastic control provides a particularly robust theoretical and operational framework for portfolio management in uncertain environments. By integrating probabilistic modeling of market dynamics - whether Brownian motion, jump processes or Lévy models - with dynamic optimization techniques (stochastic programming, Hamilton-Jacobi-Bellman equations, game theory), it becomes possible to develop robust, adaptive and mathematically sound investment strategies.

The tools developed in this work - from expected utility maximization to risk minimization, via constraint management, Bayesian approaches, or risk-adjusted performance measures - provide investors with a rigorous basis for informed, proactive asset allocation, capable of better anticipating market hazards and managing risk structurally.

This approach has demonstrated its operational relevance in a number of practical contexts:

- Pension funds, where securing long-term commitments is paramount,
- Commodities trading companies, faced with extreme volatility and logistical constraints,
- Algorithmic investing, where speed, adaptability and automation are essential.

Admittedly, there are still limitations - not least the dependence on modeled assumptions, the complexity of implementation, and the challenges of model calibration. However, the prospects opened up by the integration of advanced technologies, such as reinforcement learning, hybrid models combining artificial intelligence and finance, or the integration of ESG criteria into optimization functions, suggest the emergence of a more efficient, resilient and responsible finance.

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In this context, the increasing regulatory scrutiny surrounding algorithmic trading and risk transparency, as well as the growing demand for ESG-integrated strategies, highlight the need to align mathematical models with broader societal and compliance expectations. Stochastic control offers the flexibility to incorporate such multi-dimensional constraints, enabling asset managers to meet both performance goals and responsible investment standards.

This work thus lays the methodological foundations for rethinking classic capital and risk management paradigms, in an era of systemic complexity, digitalization, and sustainability imperatives.

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